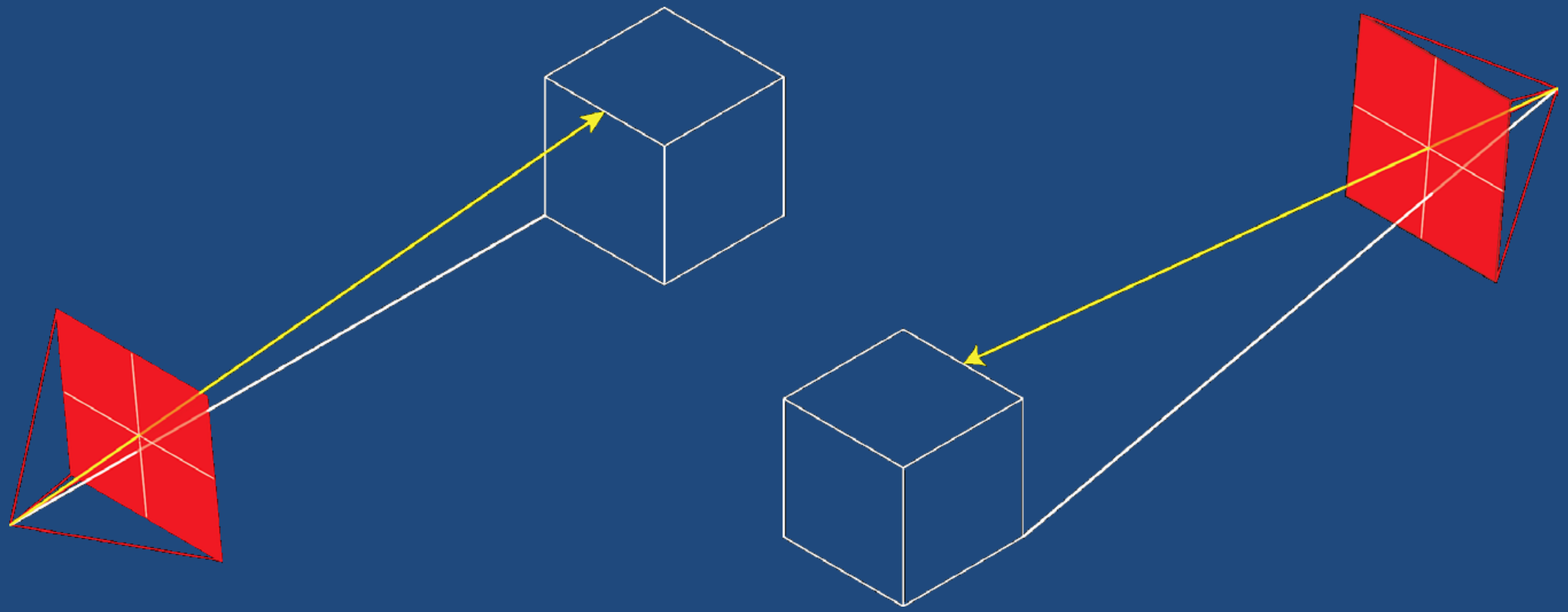
Simple Perspective Transformation - Example

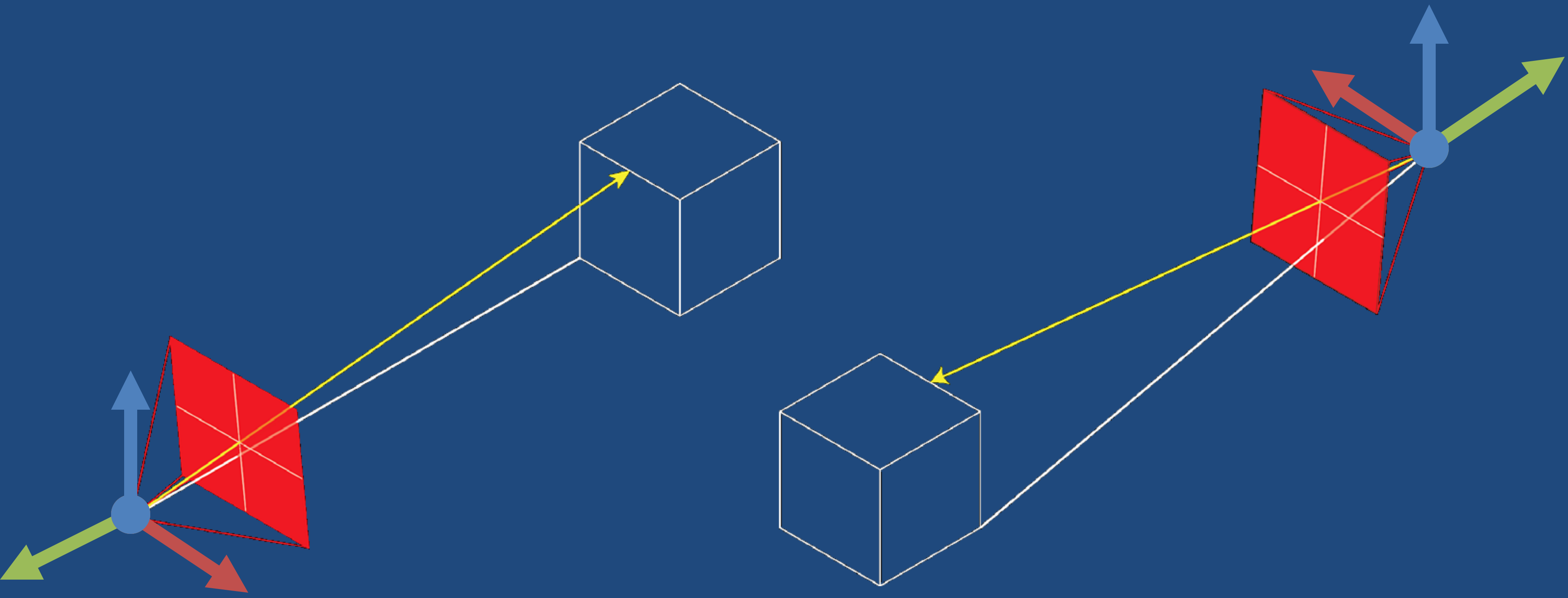


Transforming into Eye Space

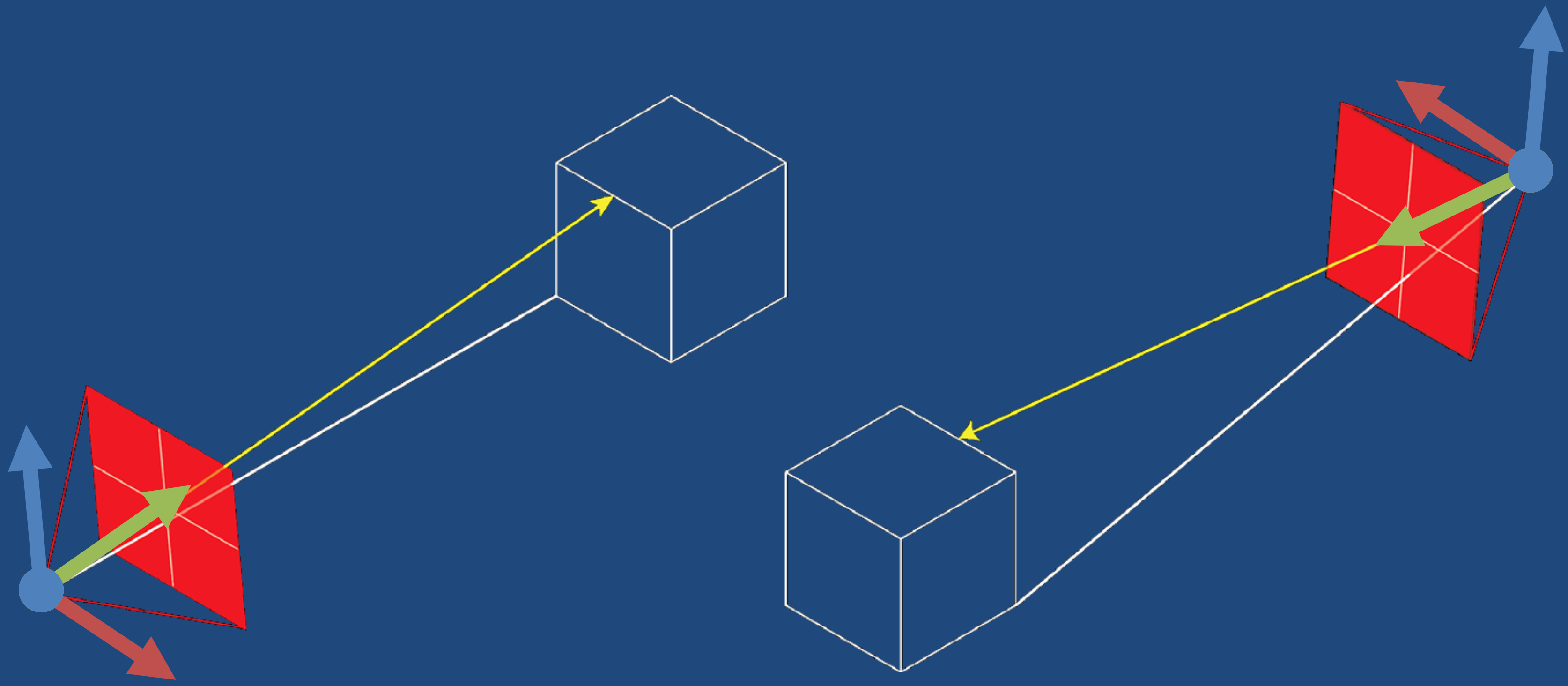
- Change of basis (axis) vectors
- Right (x), Up (y), Forward (z)

$$\begin{bmatrix} right_x & up_x & forward_x & -x \\ right_y & up_y & forward_y & -y \\ right_z & up_z & forward_z & -z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

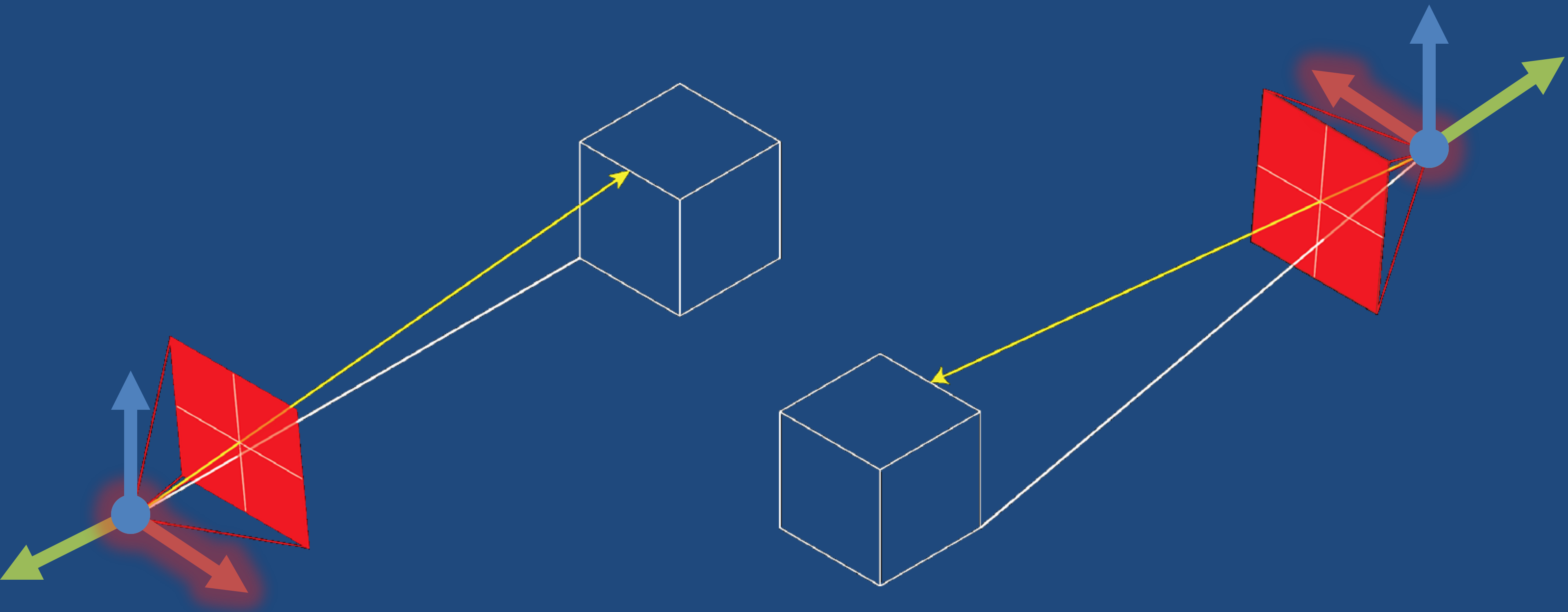
Change of Basis – World Space



Change of Basis – Eye Space

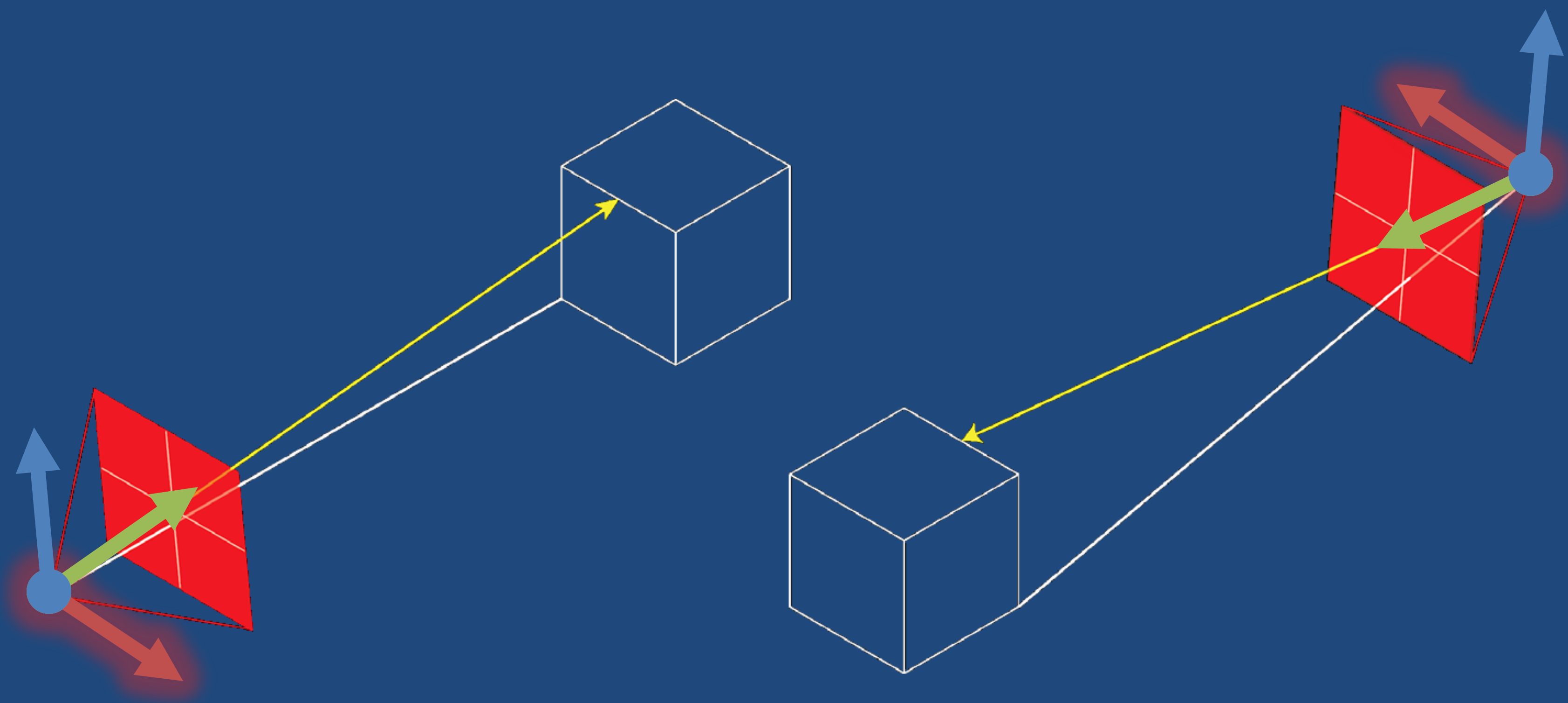


Change of Basis – World X axis



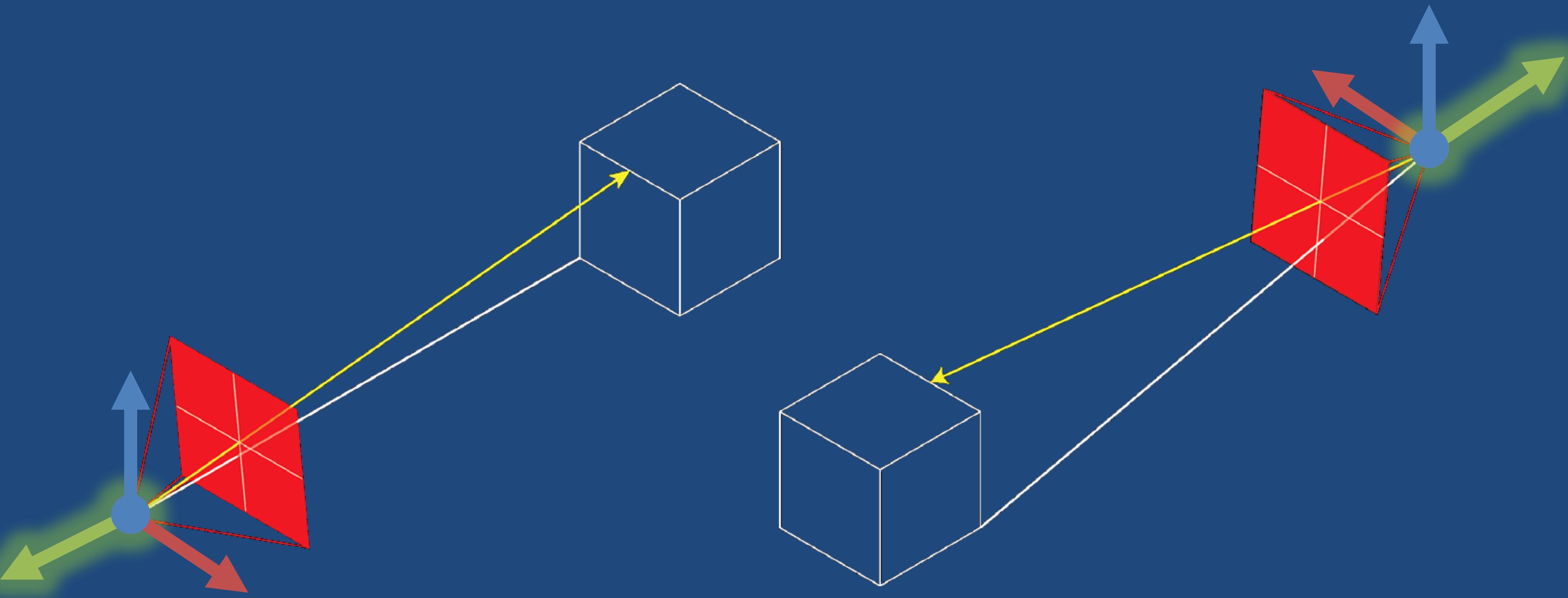
$$\langle 1, 0, 0 \rangle$$

Change of Basis – Eye X axis



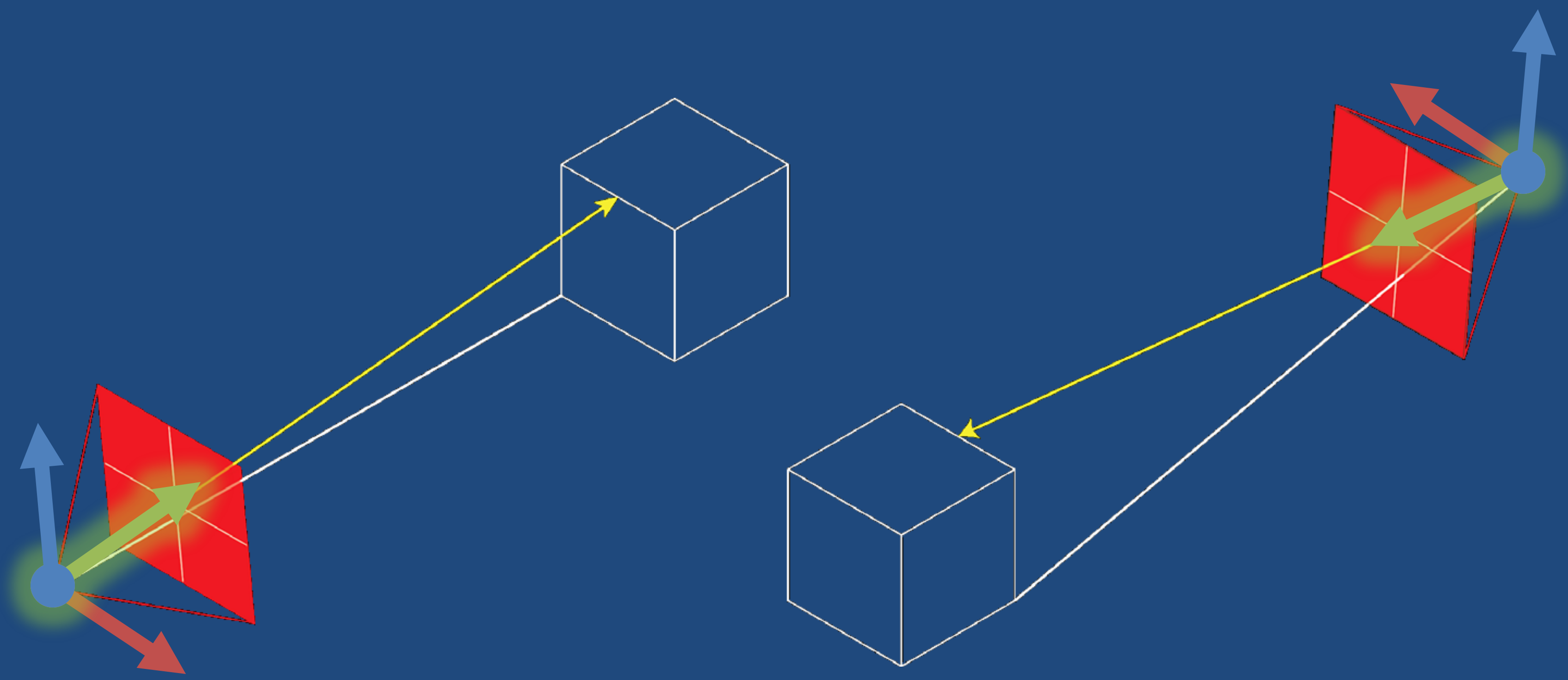
$$\langle 1, 0, 0 \rangle$$

Change of Basis – World Z axis



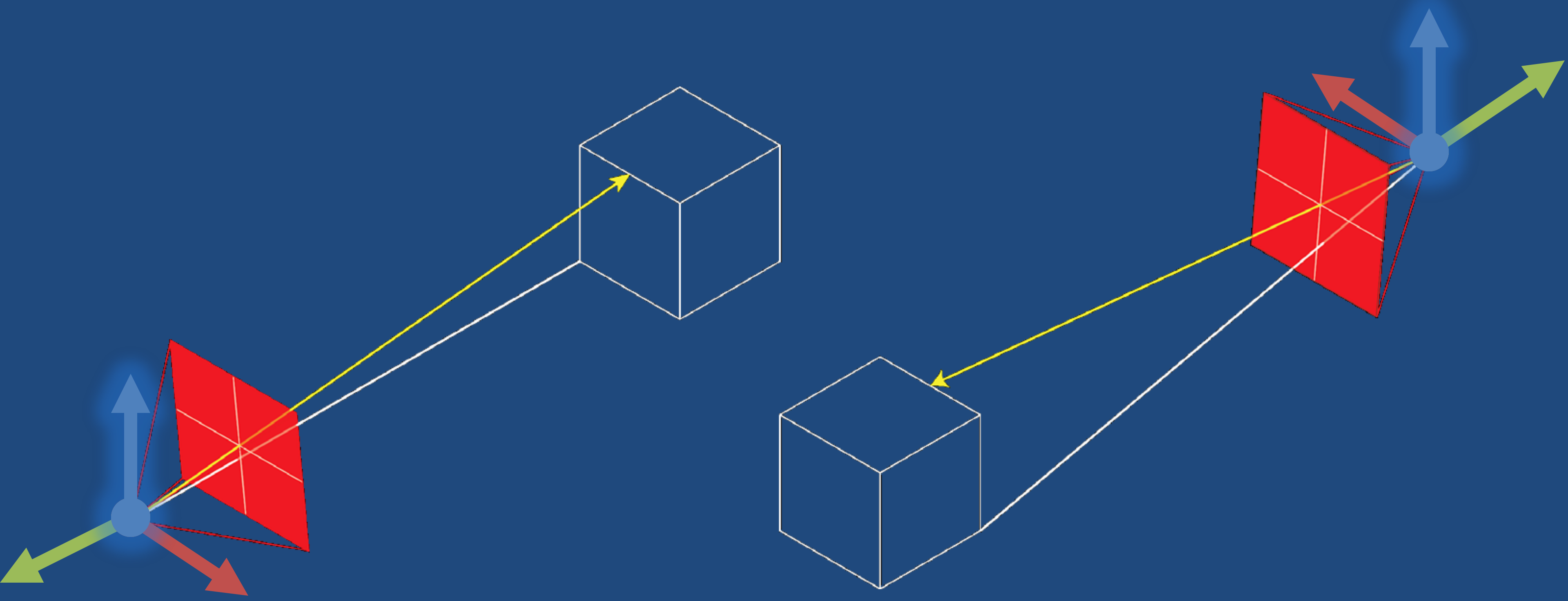
$$\langle 0, 0, 1 \rangle$$

Change of Basis – Eye Z axis



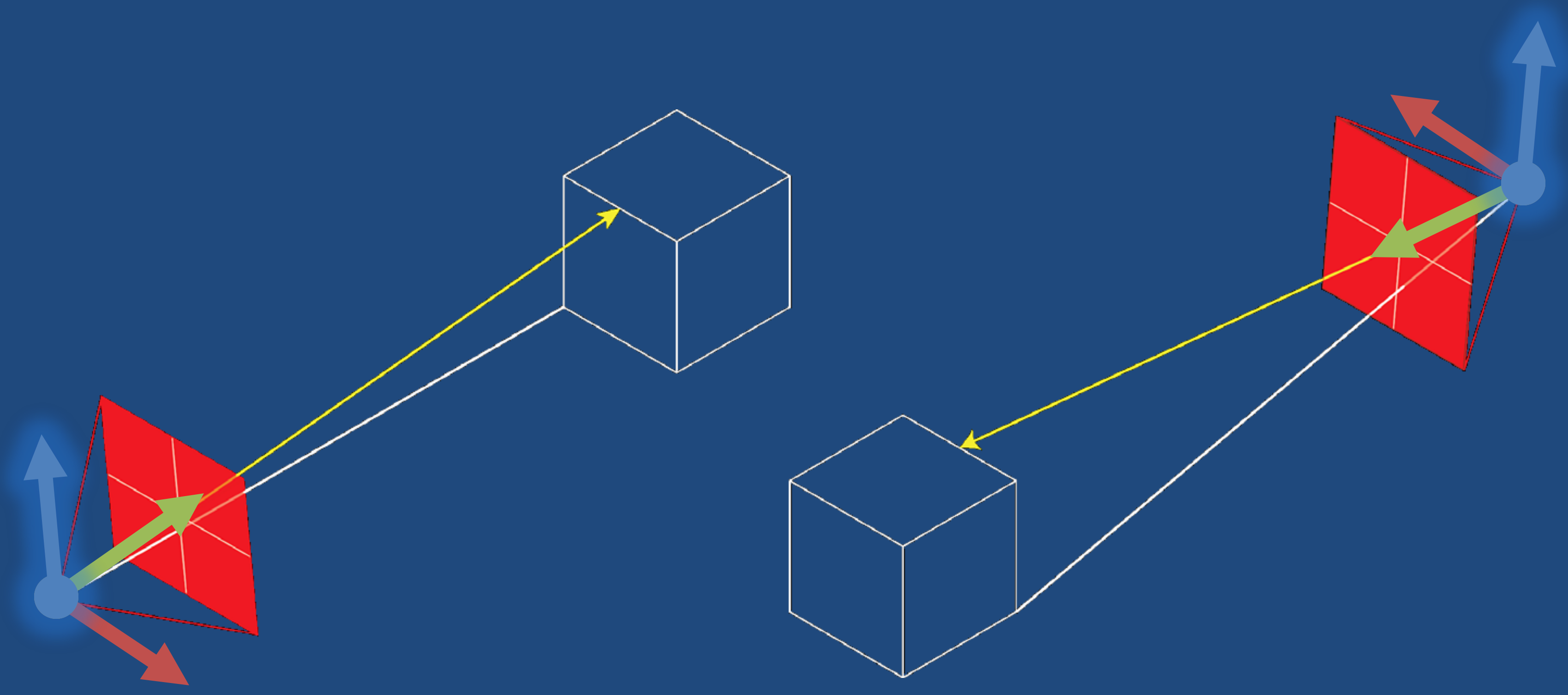
$$\langle 0, 1, -10 \rangle$$

Change of Basis – World Y axis



$$\langle 0, 1, 0 \rangle$$

Change of Basis – Eye Y axis



$$\langle 0, 10, 1 \rangle$$

Transforming into Eye Space

- Change of basis (axis) vectors
- Right (x), Up (y), Forward (z)
- Normalize new basis vectors (unit vectors)

$$\vec{u} = \frac{v}{|v|}$$

e.g.

$$v = \langle 2, 6, 3 \rangle, |v| = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{49} = 7$$

$$\vec{u} = \frac{\langle 2, 6, 3 \rangle}{7} = \left\langle \mathbf{\frac{2}{7}}, \mathbf{\frac{6}{7}}, \mathbf{\frac{3}{7}} \right\rangle$$

Transforming into Eye Space

- Normalize new basis vectors (unit vectors)

$$\vec{u} = \frac{v}{|v|}$$

$$\vec{u}_x = \frac{\langle 1, 0, 0 \rangle}{|\langle 1, 0, 0 \rangle|} = \langle \mathbf{1}, \mathbf{0}, \mathbf{0} \rangle$$

$$\vec{u}_y = \frac{\langle 0, 10, 1 \rangle}{|\langle 0, 10, 1 \rangle|} = \frac{\langle 0, 10, 1 \rangle}{\sqrt{101}} = \left\langle \mathbf{0}, \frac{\mathbf{10}}{\sqrt{\mathbf{101}}}, \frac{\mathbf{1}}{\sqrt{\mathbf{101}}} \right\rangle$$

$$\vec{u}_z = \frac{\langle 0, 1, -10 \rangle}{|\langle 0, 1, -10 \rangle|} = \frac{\langle 0, 1, -10 \rangle}{\sqrt{101}} = \left\langle \mathbf{0}, \frac{\mathbf{1}}{\sqrt{\mathbf{101}}}, \frac{\mathbf{-10}}{\sqrt{\mathbf{101}}} \right\rangle$$

Transforming into Eye Space

- Normalize new basis vectors (unit vectors)

$$\vec{u} = \frac{v}{|v|}$$

$$\vec{u}_x = \langle \mathbf{1}, \mathbf{0}, \mathbf{0} \rangle$$

$$\vec{u}_y = \left\langle \mathbf{0}, \frac{\mathbf{10}}{\sqrt{\mathbf{101}}}, \frac{\mathbf{1}}{\sqrt{\mathbf{101}}} \right\rangle = \langle \mathbf{0}, \mathbf{0.995}, \mathbf{0.100} \rangle$$

$$\vec{u}_z = \left\langle \mathbf{0}, \frac{\mathbf{1}}{\sqrt{\mathbf{101}}}, \frac{-\mathbf{10}}{\sqrt{\mathbf{101}}} \right\rangle = \langle \mathbf{0}, \mathbf{0.100}, -\mathbf{0.995} \rangle$$

Transforming into Eye Space

$$\begin{bmatrix} right_x & up_x & forward_x & -x \\ right_y & up_y & forward_y & -y \\ right_z & up_z & forward_z & -z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.995 & 0.100 & 0 \\ 0 & 0.100 & -0.995 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transforming into Eye Space

- Multiply the point by this transformation matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.995 & 0.100 & 0 \\ 0 & 0.100 & -0.995 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ -10 \\ 1 \end{bmatrix}$$

Transforming into Eye Space

- Multiply the point by this transformation matrix

$$\begin{bmatrix} -1 + 0 + 0 + 0 \\ 0 - 0.995 - 1.00 + 0 \\ 0 - 0.100 + 9.95 + 0 \\ 0 + 0 + 0 + 1 \end{bmatrix}$$

Transforming into Eye Space

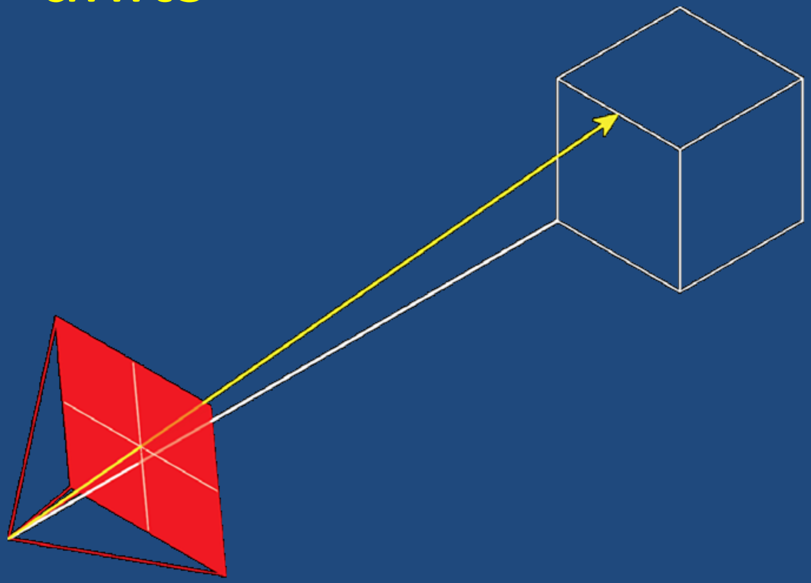
- Multiply the point by this transformation matrix

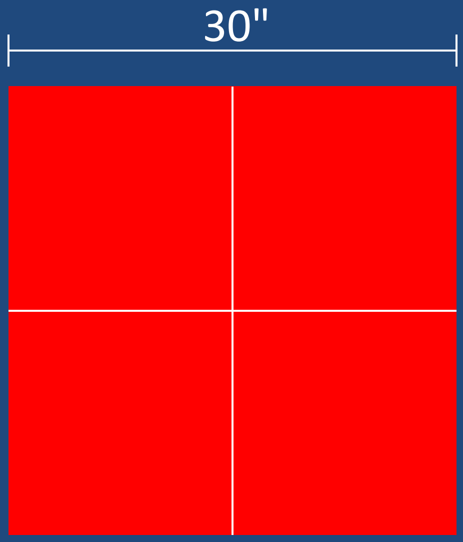
$$\begin{bmatrix} -1 \\ -1.99 \\ 9.85 \\ 1 \end{bmatrix}$$

$$X_e = -1$$
$$Y_e = -1.99$$
$$Z_e = 9.85$$

Screen Space

- Projecting world onto screen
- Converting between world units and screen units

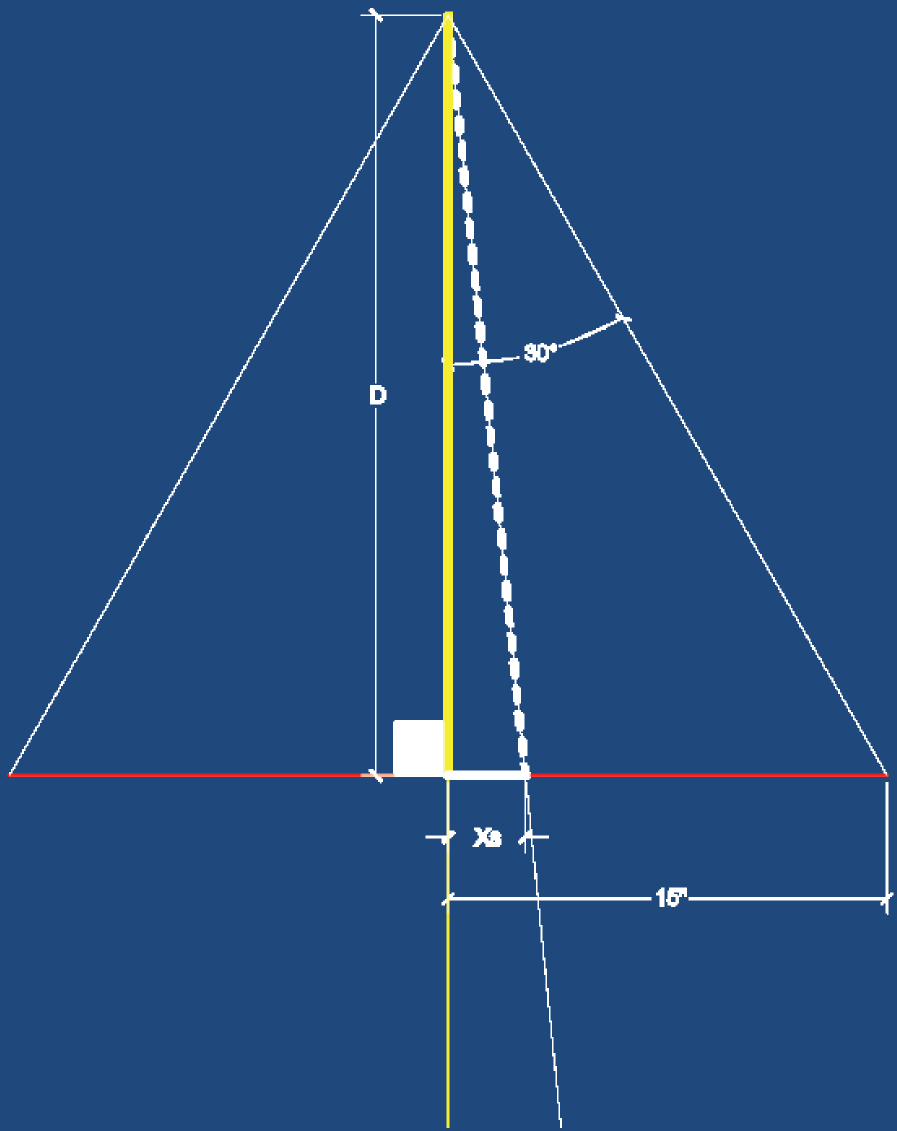




Screen Space – X axis

$$\tan(30) = \frac{15''}{D}$$

$$D = \frac{15''}{\tan(30)} = 25.98''$$



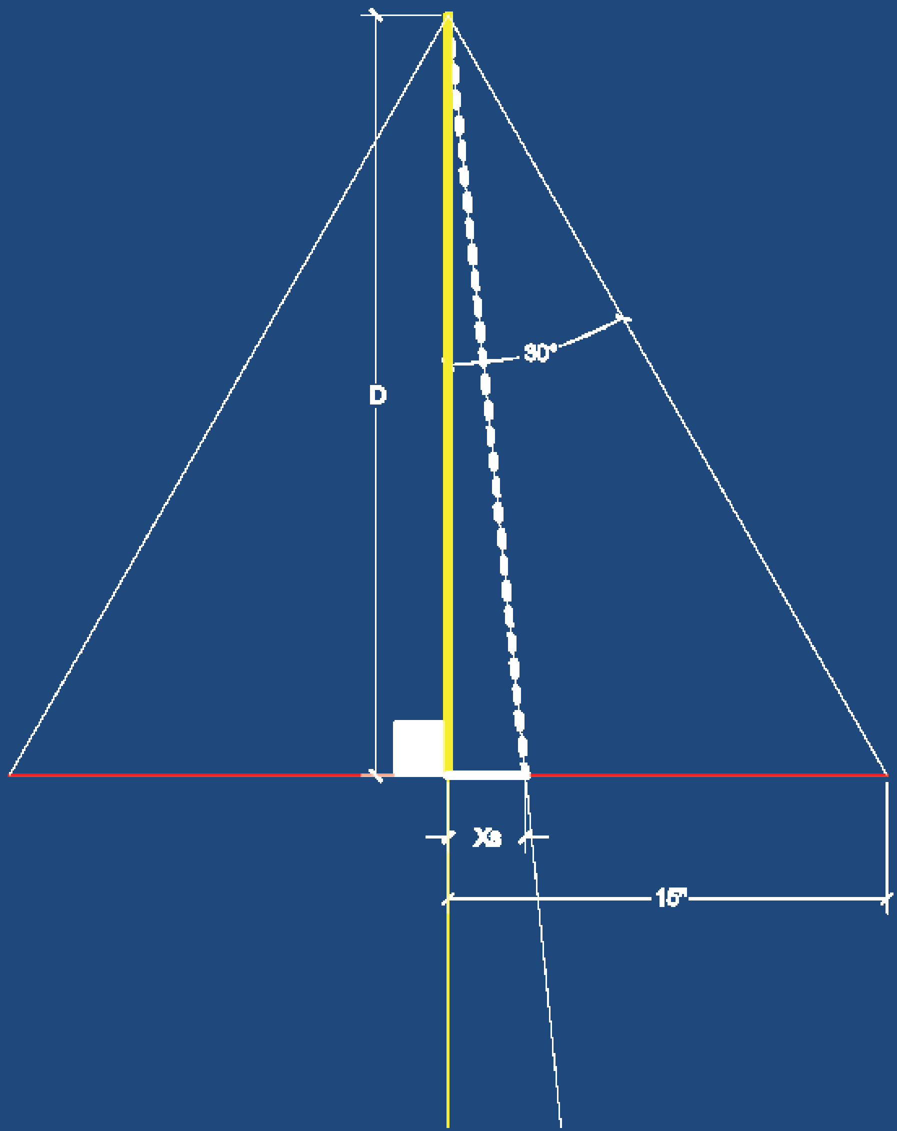
Screen Space – X axis

$$D = 25.98''$$

$$\frac{X_s}{D} = \frac{X_e}{Z_e}$$

$$\frac{X_s}{25.98} = \frac{1.00}{9.85}$$

$$X_s = 2.64''$$



Screen Space – Y axis

$$D = 25.98"$$

$$\frac{Y_s}{D} = \frac{Y_e}{Z_e}$$

$$\frac{Y_s}{25.98} = \frac{-1.99}{9.85}$$

$$Y_s = -5.25"$$